Math 426/529 Project

Due: December 9, 2024 at 11:59pm. Please submit by e-mailing your project to the instructor.

Basic Requirements: The project for this course involves reading a research paper and writing a summary in your own words.[1] Please **cite sources** properly, including figures (i.e. pictures, tables, etc). Please typeset your project, preferably using LaTeX. This is an **individual** project, not a group project. You can discuss your project with other students for the purposes of learning from each other; however, you must complete your project yourself.

Please make it clear which paper your project is based on. Give your project a title which is distinct from the title of the paper it is based on. An abstract is not necessary. Your project should start with an introduction which provides motivation, clearly explains the main results of the paper puts them in context with other work in the area.

Length: The project is worth a decent part of your grade, and so you should put a fair amount work into it, but that does not mean that it needs to be long. Quality is more important than quantity. The important thing is to communicate the ideas in an interesting way which demonstrates your understanding. As a rough guide, aim for 5 to 15 pages. That is already a wide range, and it can be shorter or longer if you wish. A longer project is not necessarily a better one.

Tips: The papers vary in length (quite substantially). For longer papers, it is probably best to omit some of the results or proofs. Alternatively, you may want to replace long proofs with a high level "sketch" that captures the main intuition of the argument without all of the details. This can be a great way to demonstrate your understanding.

For shorter papers, you may want to expand the introduction beyond what is in the paper to provide more context. This can be done by consulting other related papers (e.g. papers that your project paper references). You may choose to "flesh out" the details of proofs in greater detail than the paper does. For example, replacing instances of "clearly" with the full argument. This can be a great way to demonstrate your understanding. Don't worry if you end up with a project that is longer than the paper it is based on.

Optionally, you may choose to discuss the relationship between the result of the paper and the topics covered in the course. This will be more appropriate for some projects than others. I encourage you to make the narrative engaging, but am aware that that is hard to do when you are trying to convey complex ideas.

For students using LaTeX, you may want to consider using MathSciNet (link below) to copy/paste BibTeX entries for papers that you are citing. It is faster than writing the whole bibliography out yourself! To do so, navigate to the MathSciNet page for the paper you're citing, click the "Select alternative format" box and select BibTeX.

Topic: A list of available project topics is included below. The topics have been organized roughly based on which chapter of the notes they are most closely related to. Some topics which are not directly related to the lecture notes are also included. You may also come up with your own topic, but you should e-mail it to me for approval before starting. You may not choose a topic that you are already reading, or have read in the past, for the purposes of getting credit in another course.

Once you have selected a topic, **e-mail the instructor** with your choice. There is a limit of **two students** per topic; first come, first served!

Links have been provided to preprints of papers, when possible. Most of the published articles should be accessible via MathSciNet: `https://ezproxy.library.uvic.ca/login?url=http://www.ams.org/mathscinet/`. For some articles, it may be help to sign into the UVic VPN when trying to get access if you are not on campus: `https://www.uvic.ca/systems/services/internettelephone/remoteaccess/`.

---

[1] If you do use someone else's words, then quote them!

Chapter 1

1. E. Raty. A grid generalisation of the Kruskal-Katona theorem. Arxiv: `https://arxiv.org/abs/1908.02253`.

2. D. Duffus, D. Howerd and I. Leader. The Width of Downsets. Arxiv: `https://arxiv.org/abs/1710.05714`.

3. J. R. Johnson, I. Leader and E. Long. Correlation for permutations. Arxiv: `https://arxiv.org/abs/1909.03770`.

4. B. Janzer. Projections of antichains. Arxiv: `https://arxiv.org/abs/1911.12108`.

5. B. Janzer. A note on antichains in the continuous cube. Arxiv: `https://arxiv.org/abs/1911.03421`.

6. B. Sudakov, I. Tomon and A. Zs. Wagner. Infinite Sperner's theorem. Arxiv: `https://arxiv.org/abs/2008.04804`.

7. B. Sudakov, I. Tomon and A. Zs. Wagner. Uniform chain decompositions and applications. Arxiv: `https://arxiv.org/abs/1911.09533`.

8. W. Samotij. Subsets of posets minimising the number of chains. *Trans. Amer. Math. Soc.* 371 (2019), no. 10, 7259–7274. Arxiv: `https://arxiv.org/abs/1708.02436`.

9. J. Long and A. Zs. Wagner. The largest projective cube-free subsets of $\mathbb{Z}_{2^n}$. *European J. Combin.* 81 (2019), 156–171.

10. S. Das, W. Gan and B. Sudakov. Sperner's theorem and a problem of Erdős, Katona and Kleitman. *Combin. Probab. Comput.* 24 (2015), no. 4, 585–608. Arxiv: `https://arxiv.org/abs/1302.5210v3`.

11. B. Bollobás, G. Brightwell and R. Morris. Shadows of ordered graphs. *J. Combin. Theory Ser. A* 118 (2011), no. 3, 729–747. Arxiv: `https://arxiv.org/abs/0906.3724`.

12. V. Gruslys, S. Letzter and N. Morrison. Hypergraph Lagrangians I: the Frankl–Füredi conjecture is false. *To appear in Adv. in Math.*. 24 pages. Arxiv: `https://arxiv.org/abs/1807.00793`.

13. S. G. Z. Smith and I. Tomon. The poset on connected graphs is Sperner. *J. Combin. Theory Ser. A* 150 (2017), 162–181. Arxiv: `https://arxiv.org/abs/1511.08246`.

14. R. E. Canfield. On a problem of Rota. *Adv. in Math.* 29 (1978), no. 1, 1–10.

15. H. Spink. Orthogonal symmetric chain decompositions of hypercubes. *SIAM J. Discrete Math.* 33 (2019), no. 2, 910–932. Arxiv: `https://arxiv.org/abs/1706.08545`.

16. A. S. Bandeira, A. Ferber and M. Kwan. Resilience for the Littlewood–Offord problem. *Adv. Math.* 319 (2017), 292–312. Arxiv: `https://arxiv.org/abs/1609.08136`.

17. T. Tao and V. Vu. The Littlewood–Offord problem in high dimensions and a conjecture of Frankl and Füredi. *Combinatorica* 32 (2012), no. 3, 363–372.

18. A. J. W. Hilton. A theorem on finite sets. *Quart. J. Math. Oxford Ser.* (2) 27 (1976), no. 105, 33–36. See also Kleitman's review of this paper on MathSciNet for another proof!

Chapter 2

19. M. Bucić, S. Letzter, B. Sudakov and T. Tran. Turán numbers of sunflowers. Arxiv: https://arxiv.org/abs/1801.05471.

20. D. Ellis, G. Kalai and B. Narayanan. On symmetric intersecting families. Arxiv: https://arxiv.org/abs/1702.02607.

21. D. Bradač, M. Bucić and B. Sudakov. Turán numbers of sunflowers. Arxiv: https://arxiv.org/abs/2110.11319.

22. S. Cambie, J. Kim, H. Liu and T. Tran. A proof of Frankl's conjecture on cross-union families. Arxiv: https://arxiv.org/abs/2202.10365.

23. K. Hendry, B. Lund, C. Tompkins and T. Tran. Maximal 3-wise intersecting families. Arxiv: https://arxiv.org/abs/2110.12708.

24. E. Friedgut. A Katona-type proof of an Erdős–Ko–Rado-type theorem. *J. Combin. Theory Ser. A* 111 (2005), no. 2, 239–244.

25. B. Bollobás and I. Leader. Set systems with few disjoint pairs. *Combinatorica* 23 (2003), no. 4, 559–570.

26. M. Bucić, S. Glock and B. Sudakov. The intersection spectrum of 3-chromatic intersecting hypergraphs. 9 pages. Arxiv: https://arxiv.org/abs/2010.00495.

27. J. Kahn and G. Kalai. A counterexample to Borsuk's conjecture. *Bull. Amer. Math. Soc. (N.S.)* 29 (1993), no. 1, 60–62.

28. L. Lovász. Kneser's conjecture, chromatic number, and homotopy. *J. Combin. Theory Ser. A* 25 (1978), no. 3, 319–324.

29. M. Chudnovsky, R. Kim, C.-H. Liu, P. Seymour, and S. Thomassé. Domination in tournaments. *J. Combin. Theory Ser. B* 130 (2018), 98–113.

30. N. Alon, J. Balogh, B. Bollobás, R. Morris. The structure of almost all graphs in a hereditary property. *J. Combin. Theory Ser. B* 101 (2011), no. 2, 85–110. Arxiv: https://arxiv.org/abs/0905.1942.

31. N. Frankl, S. Kiselev, A. Kupavskii and B. Patkós. VC-saturated set systems. Arxiv: https://arxiv.org/abs/2005.12545.

Chapter 3

32. D. Conlon. Extremal numbers of cycles revisited. Arxiv: https://arxiv.org/abs/2011.11064.

33. D. Bradač, O. Janzer, B. Sudakov and I. Tomon. The Turán number of the grid. Arxiv: https://arxiv.org/abs/2203.05485.

34. J. Balogh, F. C. Clemen and B. Lidický. Hypergraph Turán Problems in ℓ_2-Norm. Arxiv: https://arxiv.org/abs/2108.10406.

35. J. Balogh, F. C. Clemen and B. Lidický. 10 Problems for Partitions of Triangle-free Graphs. Arxiv: https://arxiv.org/abs/2203.15764.

36. O. Janzer and B. Sudakov. Resolution of the Erdős-Sauer problem on regular subgraphs. Arxiv: https://arxiv.org/abs/2204.12455.

37. J. Balogh, F. C. Clemen and H. Luo. Non-degenerate Hypergraphs with Exponentially Many Extremal Constructions. Arxiv: https://arxiv.org/abs/2208.00652.

38. B. Janzer. Large hypergraphs without tight cycles. Arxiv: `https://arxiv.org/abs/2012.07726`.

39. D. Conlon and O. Janzer. Rational exponents near two. Arxiv: `https://arxiv.org/abs/2203.03375`.

40. O. Janzer. Disproof of a conjecture of Erdős and Simonovits on the Turán number of graphs with minimum degree 3. Arxiv: `https://arxiv.org/abs/2109.06110`.

41. P. Keevash, I. Leader, J. Long and A. Zs. Wagner. The extremal number of Venn diagrams. Arxiv: `https://arxiv.org/abs/1911.00487`.

42. D. Ellis and D. King. Lower bounds for the Turán densities of daisies. Arxiv: `https://arxiv.org/abs/2204.08930`.

43. A. Grzesik, O. Janzer and Z. L. Nagy. The Turán number of blow-ups of trees. Arxiv: `https://arxiv.org/abs/1904.07219`.

44. J. Verstraëte. On arithmetic progressions of cycle lengths in graphs, *Combin. Probab. Comput.* 9 (2000), no. 4, 369–373. Arxiv: `https://arxiv.org/abs/math/0204222`.

45. B. Sudakov and J. Verstraëte. The extremal function for cycles of length ℓ mod k. *Electron. J. Combin.* 24 (2017), no. 1, Paper No. 1.7, 8 pp. Arxiv: `https://arxiv.org/abs/1606.08532`.

46. B. Sudakov and J. Verstraëte. Cycle lengths in sparse graphs. *Combinatorica* 28 (2008), no. 3, 357–372. Arxiv: `https://arxiv.org/abs/0707.2117`.

47. B. Bukh and D. Conlon. Rational exponents in extremal graph theory. *J. Eur. Math. Soc. (JEMS)* 20 (2018), no. 7, 1747–1757. Arxiv: `https://arxiv.org/abs/1506.06406`.

48. J. Pach and G. Tardos. Forbidden paths and cycles in ordered graphs and matrices. *Israel J. Math.* 155 (2006), 359–380.

49. J. Kollár, L. Rónyai and T. Szabó. Norm-graphs and bipartite Turán numbers. *Combinatorica* 16 (1996), no. 3, 399–406.

50. N. Alon, L. Rónyai and T. Szabó. Norm-graphs: variations and applications. *J. Combin. Theory Ser. B* 76 (1999), no. 2, 280–290.

51. A. Sah, M. Sawhney and Y. Zhao. Paths of given length in tournaments. 4 pages. Arxiv: `https://arxiv.org/abs/2012.00262`.

52. N. Alon and C. Shikhelman. Many T copies in H-free graphs. *J. Combin. Theory Ser. B* 121 (2016), 146–172. Arxiv: `https://arxiv.org/abs/1409.4192`.

53. D. Korándi, G. Tardos, I. Tomon and C. Weidert. On the Turán number of ordered forests. *J. Combin. Theory Ser. A* 165 (2019), 32–43. Arxiv: `https://arxiv.org/abs/1711.07723`.

Chapter 4

54. D. Conlon, S. Luo and M. Tyomkyn. Ramsey numbers of books and quasirandomness. Arxiv: `https://arxiv.org/abs/2204.11360`.

55. D. Conlon, J. Fox and Y. Wigderson. Ramsey numbers of books and quasirandomness. Arxiv: `https://arxiv.org/abs/2001.00407`.

56. J. He, J. Ma, T. Yang. Stability and supersaturation of 4-cycles. 41 pages. Arxiv: `https://arxiv.org/abs/1912.00986`.

57. A. Thomason. Graph products and monochromatic multiplicities. *Combinatorica* 17 (1997), no. 1, 125–134.

58. D. Korándi, A. Roberts and A. Scott. Exact Stability for Turán's Theorem. 16 pages. Arxiv: https://arxiv.org/abs/2004.10685.

59. J. Cutler, J. D. Nir and A. J. Radcliffe. Supersaturation for subgraph counts. 17 pages. Arxiv: https://arxiv.org/abs/1903.08059.

60. A. Halfpap, C. Palmer. On supersaturation and stability for generalized Turán problems. 8 pages. Arxiv: https://arxiv.org/abs/1909.13043.

61. S. Berger, J. Lee and M. Schacht. Odd cycles in subgraphs of sparse pseudorandom graphs. 14 pages. Arxiv: https://arxiv.org/abs/1906.05100.

62. O. Janzer and B. Sudakov. On the Turán number of the hypercube. Arxiv: https://arxiv.org/abs/2211.02015.

Chapter 5

63. D. Conlon, J. Fox, B. Sudakov and Y. Zhao. Which graphs can be counted in C_4-free graphs? Arxiv: https://arxiv.org/abs/2106.03261.

64. D. Conlon, J. Fox, B. Sudakov and Y. Zhao. The regularity method for graphs with few 4-cycles. Arxiv: https://arxiv.org/abs/2004.10180.

65. J. Fox and Y. Zhao. Removal lemmas and approximate homomorphisms. Arxiv: https://arxiv.org/abs/2104.11626.

66. G. Moshkovitz and A. Shapira. A short proof of Gowers' lower bound for the regularity lemma. *Combinatorica* 36 (2016), no. 2, 187–194. Arxiv: https://arxiv.org/abs/1308.5352.

67. N. Alon, J. Balogh, P. Keevash and B. Sudakov. The number of edge colorings with no monochromatic cliques. *J. London Math. Soc.* (2) 70 (2004), no. 2, 273–288.

68. D. Conlon and J. Fox. Bounds for graph regularity and removal lemmas. *Geom. Funct. Anal.* 22 (2012), no. 5, 1191–1256. Arxiv: https://arxiv.org/abs/1606.01043.

69. N. Alon, A. Moitra and B. Sudakov. Nearly complete graphs decomposable into large induced matchings and their applications. *J. Eur. Math. Soc. (JEMS)* 15 (2013), no. 5, 1575–1596. Arxiv: https://arxiv.org/abs/1111.0253.

70. N. Alon, A. Shapira and U. Stav. Can a graph have distinct regular partitions? *SIAM J. Discrete Math.* 23 (2008/09), no. 1, 278–287. Preprint available on Asaf Shapira's webpage: http://www.math.tau.ac.il/~asafico/regiso.pdf.

71. J. Fox and L. M. Lovász. A tight lower bound for Szemerédi's regularity lemma. *Combinatorica* 37 (2017), no. 5, 911–951. Arxiv: https://arxiv.org/abs/1403.1768.

72. J. Balogh, S. Jiang and H. Luo. On the maximum number of r-cliques in graphs free of complete r-partite subgraphs. Arxiv: https://arxiv.org/abs/2402.16818v1.

73. J. Balogh, V. Magnan, C. Palmer. Generalized Ramsey–Turán Numbers. Arxiv: https://arxiv.org/abs/2405.01804.

74. J. Gao, S. Jiang, H. Liu, M. Sankar. Generalized Ramsey–Turán density for cliques. Arxiv: https://arxiv.org/abs/2403.12919.

Chapter 6

75. D. Conlon. A sequence of triangle-free pseudorandom graphs. *Combin. Probab. Comput.* 26 (2017), no. 2, 195–200.

76. Y. Zhao. The number of independent sets in a regular graph. *Combin. Probab. Comput.* 19 (2010), no. 2, 315–320. Arxiv: `https://arxiv.org/abs/0909.3354`.

77. E. Davies, M. Jenssen, W. Perkins and B. Roberts. On the average size of independent sets in triangle-free graphs. *Proc. Amer. Math. Soc.* 146 (2018), no. 1, 111–124.

78. J. Balogh and Š. Petříčková. The number of the maximal triangle-free graphs. *Bull. Lond. Math. Soc.* 46 (2014), no. 5, 1003–1006. Arxiv: `https://arxiv.org/abs/1409.8123`.

79. N. Alon. The maximum number of Hamiltonian paths in tournaments. *Combinatorica* 10 (1990), no. 4, 319–324.

80. D. Galvin. Independent sets in the discrete hypercube. Arxiv: `https://arxiv.org/abs/1901.01991`.

81. M. Jenssen and W. Perkins. Independent sets in the hypercube revisited. Arxiv: `https://arxiv.org/abs/1907.00862`.

82. S. Mattheus and J. Verstraëte. The asymptotics of $r(4,t)$. Arxiv: `https://arxiv.org/abs/2306.04007`.

83. I. Balla, O. Janzer and B. Sudakov. On MaxCut and the Lovász theta function. Arxiv: `https://arxiv.org/abs/2305.18252`.

84. D. G. Zhu. An improved lower bound on the Shannon capacities of complements of odd cycles. Arxiv: `https://arxiv.org/abs/2402.10025`.

85. G. Simonyi. Shannon capacity and the categorical product. Arxiv: `https://arxiv.org/abs/1911.00944v1`.

86. T. Bohman. A limit theorem for the shannon capacities of odd cycles I. Link: `https://www.ams.org/journals/proc/2003-131-11/S0002-9939-03-06495-5/S0002-9939-03-06495-5.pdf`.

Chapter 7

87. G. Blekherman, A. Raymond and F. Wei. Undecidability of polynomial inequalities in weighted graph homomorphism densities. Arxiv: `https://arxiv.org/abs/2207.12378`.

88. N. Linial and Z. Luria. An upper bound on the number of high-dimensional permutations. *Combinatorica* 34 (2014), no. 4, 471–486. Arxiv: `https://arxiv.org/abs/1106.0649`.

89. J. Lee. On some graph densities in locally dense graphs. Arxiv: `https://arxiv.org/abs/1707.02916`.

90. A. Grzesik, J. Lee, B. Lidický, J. Volec. On tripartite common graphs. Arxiv: `https://arxiv.org/abs/2012.02057`.

91. J. Kahn and J. Park. The number of 4-colorings of the Hamming cube. Arxiv: `https://arxiv.org/abs/1808.01152`.

Other Topics

92. J. Haslegrave, J. Hyde, J. Kim and H. Liu. Ramsey numbers of cycles versus general graphs. Arxiv: `https://arxiv.org/abs/2112.03893`.

93. N. C. Behague. Hypergraph saturation irregularities. *Electron. J. Combin.* 25 (2018), no. 2, Paper No. 2.11, 13 pp. Arxiv: `https://arxiv.org/abs/1803.05799`.

94. W. Gan, D. Korándi and B. Sudakov. $K_{s,t}$-saturated bipartite graphs. *European J. Combin.* 45 (2015), 12–20. Arxiv: `https://arxiv.org/abs/1402.2471`.

95. S. Norin and Y. Yuditsky. Erdős–Szekeres Without Induction. *Discrete Comput. Geom.* 55 (2016), no. 4, 963–971.

96. G. Blekherman and S. Patel. Threshold Graphs Maximize Homomorphism Densities. 15 pages. Arxiv: `https://arxiv.org/abs/2002.12117`.

97. J. Fox, X. He and F. Manners. A proof of Tomescu's graph coloring conjecture. *J. Combin. Theory Ser. B* 136 (2019), 204–221. Arxiv: `https://arxiv.org/abs/1712.06067`.

98. J. Fox, A. Grinshpun and J. Pach. The Erdős-Hajnal conjecture for rainbow triangles. *J. Combin. Theory Ser. B* 111 (2015), 75–125. Arxiv: `https://arxiv.org/abs/1303.2951`.

99. D. Král', S. Norin and J. Volec. A bound on the inducibility of cycles. *J. Combin. Theory Ser. A* 161 (2019), 359–363. Arxiv: `https://arxiv.org/abs/1801.01556`.

100. L. Lovász and B. Szegedy. Limits of dense graph sequences. *J. Combin. Theory Ser. B* 96 (2006), no. 6, 933–957. Arxiv: `https://arxiv.org/abs/math/0408173`.

101. C. Reiher. Counting odd cycles in locally dense graphs. *J. Combin. Theory Ser. B* 105 (2014), 1–5. Arxiv: `https://arxiv.org/abs/1604.06833`.

102. C. Groenland and T. Johnston. Intersection sizes of linear subspaces with the hypercube. *J. Combin. Theory Ser. A* 170 (2020), 105142, 14 pp. Arxiv: `https://arxiv.org/abs/1810.02729v1`.